Depth-independent cone penetration mechanism by a discrete element method (DEM)-based stress normalization approach

Su Liu and Jianfeng Wang

Abstract: A two-dimensional discrete element model of driven piles in crushable sand was developed and validated against laboratory data. Numerical experiments were conducted to investigate the effects on the pile penetration behavior of initial sample porosity, particle crushability, initial stress state, ratio of pile diameter to median particle diameter, and ratio of model width to pile diameter. A new stress normalization method is adopted to synthesize the data at different driven depths from the simulations. The normalized vertical and horizontal stress fields surrounding the pile show an invariable pattern of stress distribution, suggesting a unique penetration mechanism independent of the penetration depth. The validity of the discrete element method (DEM) simulation results is verified by comparing the stress distributions to those observed from calibration chamber tests on model pile installation in sands.

Key words: pile penetration, crushable sand, particle breakage, discrete element method (DEM) simulation, stress normalization.

Résumé : Un modèle bidimensionnel de pieux enfouis dans du sable écrasable, basé sur la méthode des éléments discrets (MED), a été développé et validé à partir des données obtenues en laboratoire. On a effectué des Expériences numériques pour étudier les effets de la porosité initiale d’échantillons, de l’écrasabilité des particules, de l’état de contrainte initial, le quotient du diamètre du pieu par le diamètre moyen des particules et quotient de la largeur du modèle par le diamètre du pieu, sur le mécanisme de pénétration du pieu. Une nouvelle méthode de normalisation des contraintes a été adoptée, permettant de synthétiser les données de simulation obtenues à différentes profondeurs d’enfouissement du pieu. Les champs de contraintes normalisées verticales et horizontales entourant le pieu montrent que les motifs de distribution des contraintes ne varient pas, ce qui met en évidence un mécanisme de pénétration unique indépendant de la profondeur de pénétration. On vérifie la validité des résultats de la simulation basée sur la MED en comparant les distributions des contraintes à celles obtenues grâce à des essais en chambre d’étalonnage effectués sur des pieux expérimentaux enfouis dans du sable. [Traduit par la Rédaction]

Mots-clés : pénétration d’un pieu, sable écrasable, rupture des particules, simulation basée sur la méthode des éléments discrets (MED), normalisation des contraintes.

Introduction

Over the past two decades, field and laboratory investigations on instrumented prototype or model piles installed in sand have made considerable contributions towards revealing the pile penetration mechanism and establishing more rational and reliable methods for pile design in sand (e.g., Lehane 1992; Randolph et al. 1994; Jardine and Chow 1996; Klotz and Coop 2001; Jardine et al. 2013a). The stresses measured directly at the tip and along the shaft of the pile in this type of study provide first-hand information for understanding a range of complex soil behaviors arising from the pile installation, including very large soil deformation (Liyanapathirana et al. 2000), localized interface shearing (White and Bolton 2004), particle breakage (Yang et al. 2010), load cycling and shaft friction degradation (White and Lehané 2004), soil creep and pile setup (Chow et al. 1998; Lim and Lehané 2014), etc.

By using highly instrumented calibration chamber tests on the driving of model displacement piles (Jardine et al. 2013a, 2013b; Yang et al. 2014), the full-field stress distribution within the sand mass surrounding the pile could also be obtained, facilitating examination of the chamber boundary effects and establishment of the linkage between penetration resistances measured in the calibration chambers and the field (Pouraghiazer et al. 2012). The experimental results of physical modeling tests provide benchmarks for theoretical and numerical modeling of the pile penetration problems.

In parallel with the experimental investigations, considerable advances have also been achieved on this topic using the discrete element method (DEM), whose advantages include full access to the particle-scale kinetic and kinematic information and the potential of being a virtual testing tool substituting physical model tests (Lobo-Guerrero and Vallejo 2005; Jiang et al. 2006; Wang et al. 2007a, 2007b; Arroyo et al. 2011; Wang and Jiang 2011; Lin and Wu 2012; Mcdowell et al. 2012; Butlanska et al. 2014; Falagush et al. 2015). In a previous related study (Wang and Zhao 2014), the authors made a detailed discrete-continuum analysis of the pile penetration behavior based on the two-dimensional (2D) DEM simulation results. The stress and strain data provided by the model were compared with experimental results from instrumented model piles by Klotz and Coop (2001) and the particle image velocimetry (PIV) measurements of soil deformation in calibration chamber tests by White and Bolton (2004), and were mainly used to demonstrate the effects of in situ stress field, initial soil density, and particle crushability on the pile penetration behavior. The current study, on the basis of that previous study, aims to further probe the penetration mechanism by deepening the stress analysis of the pile and sand mass. The goal is to unravel and present a unique penetration mechanism within a homogeneous sand mass that is independent of the pile penetration depth. The effects of two additional model-scale variables — namely, the ratio of pile diameter to median particle...
diameter and the ratio of model width to pile diameter were included in the current study to enhance the research findings.

**Numerical method**

Simulations were carried out using the PFC2D program (Itasca Consulting Group 2008). Crushable particles are simulated by agglomerates, each of which is composed of 24–30 parallel-bonded elementary disks with diameters between 0.069 and 0.278 mm (Fig. 1d). The contact between two elementary disks within an agglomerate consists of three parts: a linear stiffness model, a slip model, and a parallel-bond model (Cheng et al. 2003; Wang and Yan 2013). A parallel bond breaks if the normal or shear stress acting on the bond exceeds its corresponding bond strength. The conventional linear contact model with a slip failure mechanism will take effect after a parallel bond is broken. The advantage of the parallel-bond model over the contact-bond model is that it simulates a finite piece of cement between two disks providing a rotational resistance. More details of the particle-contact model can be found in Wang and Yan (2013).

**Numerical biaxial test**

Prior to the numerical penetration test, the behavior of the DEM materials was first characterized by means of the numerical biaxial test. The size of the specimen was 80 mm in height and 40 mm in width. The top and bottom walls were set to be frictionless. The lateral flexible membrane boundaries (Åström et al. 2000; Alonso-Marroquin and Herrmann 2002; Wang and Leung 2008; Zhou et al. 2013) were modeled by strings of small mono-sized particles connected by contact bonds. During the construction of the models of both the biaxial and penetration tests, the granular samples were prepared by depositing and compacting granulates in multiple layers to achieve a homogeneous dense state. The granular material was composed of crushable agglomerates or uncrushable disks with diameters uniformly varying between 0.6 and 1.2 mm. The input DEM parameters of both models are given in Table 1.

Figure 2 shows the deviatoric stress ratio \( \tau/\sigma \) \( (\tau = (\sigma_1 - \sigma_3)/2, \sigma = (\sigma_1 + \sigma_3)/2) \) (where \( \tau \) is the deviatoric stress, and \( \sigma_1 \) and \( \sigma_3 \) are the major and minor principal stresses, respectively) and volumetric strain versus axial strain relationships. Each test is denoted using a code of “b-c, x”, where \( b \) is the parallel bond strength (in N/m), \( c \) is the initial sample porosity, and \( x \) is the confining pressure. A maximum confining pressure of 1.0 MPa was chosen because it was close to the maximum value of confining stress encountered in penetration tests. The general trends of the stress–strain behavior from Fig. 2 include (i) reduction of the peak and critical state stress ratios with increasing confining stress for a sample with fixed bond strength and porosity, (ii) an overall upward shift of the stress–strain curve with decreasing porosity for a sample with fixed bond strength and confining stress, and (iii) a reduction of both stress ratios with decreasing bond strength for a sample with fixed porosity and confining stress. The above trends reflect the results of the competition between particle breakage, sample porosity, and confining stress in controlling the stress ratio behavior (Wang and Yan 2012). It is noted that trend (i) has led to a continuous hardening behavior (without a distinct peak) for a dense sample with a porosity of 0.13 at a confining stress of 1.0 MPa, and a corresponding volumetric contraction behavior throughout the test, which is all

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**Fig. 1.** (a) Model geometry of tests 1–5; (b) layout of groups at fixed positions, with size of each group being 0.5B × 0.5B (where B is pile diameter); (c) layout of groups at relative positions to pile tip; (d) typical agglomerate composed of parallel-bonded disks.

**Fig. 2.** Evolution of (a) stress ratio and (b) volume strain of biaxial test samples. [Colour online.]
characteristic of a loose sand due to massive particle breakage occurring during the test. In contrast, trend (iii) does not show a pronounced effect when the confining stress is too low (e.g., 0.1 MPa), making only a slight difference in the stress-strain curves of samples “2e7-0.13, 0.1 MPa” and “1e7-0.13, 0.1 MPa”, which is now due to the insignificant particle breakage induced. For all the cases, the critical state friction angle varies between 25° and 27° as the confining stress is varied.

**DEM modelling of pile penetration in sand**

**Model construction**

The DEM model of the penetration test is made up of a rectangular container filled with a well-compacted, polydispersed assembly of round particles and a model pile with a triangular tip (two inclined planes each making an angle of 60° with the horizontal) pushed gradually into the granular foundation. The friction coefficient of the model pile was set to 0.5. As illustrated in Fig. 1a, only the right half of the model is used, taking advantage of the axial symmetry of the problem. All the boundary walls are fixed. The bottom and left walls are set to be frictionless, while the right wall is frictional with a friction coefficient of 0.9 to minimize any relative slip between the particles and the wall. The full model of the granular foundation consists of two zones: a crushable zone of penetration test and an uncrushable zone of penetration test.

**Simulation program**

Seven numerical penetration tests, as summarized in Table 2, were conducted. Tests were grouped to examine different issues that influence the penetration results, such as (a) lateral boundary (tests 3 and 6), (b) pile size (tests 3 and 7), (c) initial vertical stress (tests 1, 2, and 3), (d) initial porosity (tests 3 and 4), and (e) particle crushability (tests 4 and 5). Specifically, the boundary effect and pile size effect were studied by varying the value of \(2W/B\) or \(B/d_{so}\) and \(W/H\) in test 6 and \(B/d_{so}\) (where \(d_{so}\) is the median particle size) in test 7, respectively. The varying initial vertical stress field was achieved by applying artificially raised gravitational acceleration, and the sand crushability was defined by different parallel bond strength values (\(p_{b_s}\)). The model pile is pushed monotonically into the granular foundation at a constant rate of 0.1 mm/s, which is slow enough to maintain the quasi-static loading condition throughout the test. The maximum penetration depth is 176 mm in tests 1–6 and 352 mm in test 7. The computational time using an Intel Core i7–4770K CPU at 3.50 GHz and with 16 Gb RAM was about 3 days for the regular-sized model and 28 days for the double-sized model (i.e., test 7).

**Results and discussion**

**Tip resistance curves**

Figure 3 shows the pile tip resistance, \(q_{t0}\), registered from all simulation tests. Results are categorized to show the effects of model variables on the penetration resistance in six subfigures. Figures 3a and 3b show the effects of artificial gravity levels on the tip resistance using the equivalent prototype depth (where 1 mm of depth in the model with an artificial gravity of \(g_{a}\) represents \(x\) mm of depth at prototype scale) and the actual model penetration depth, respectively. Use of the prototype depth enables examination of the variation of penetration resistance against the in situ field stress based on the model result. It is clear that the three gravity levels produce three distinct curves in the plot using the model penetration depth (Fig. 3b), but result in one unique curve when plotted against the prototype depth (Fig. 3a). Note, that the data from test 1 are excluded from Fig. 3a due to inaccurate

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**Table 1. Input parameters for DEM simulations.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Biaxial test; crushable zone of penetration test</th>
<th>Flexible boundary</th>
<th>Uncrushable zone of penetration test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of agglomerates (mm)</td>
<td>0.6–1.2</td>
<td>0.225</td>
<td>0.6–1.2</td>
</tr>
<tr>
<td>Diameter of elementary disks (mm)</td>
<td>0.069–0.278</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Density of disk (kg/m³)</td>
<td>2650</td>
<td>1000</td>
<td>2200</td>
</tr>
<tr>
<td>Normal and shear stiffnesses of disk (N/m)</td>
<td>4e⁸</td>
<td>8e⁷</td>
<td>4e⁸</td>
</tr>
<tr>
<td>Friction coefficient of disk</td>
<td>0.5</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Normal and shear parallel-bond strengths (N/m²)</td>
<td>1e⁷, 2e⁷</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Normal and shear parallel-bond stiffnesses (N/m²)</td>
<td>1.5e¹²</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Normal and shear contact-bond strengths (N)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Ratio of parallel bond radius to disk radius</td>
<td>0.5</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Table 2. Characteristics of pile penetration tests.**

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Dimensions, WxH (mm)</th>
<th>Pile diameter, B (m)</th>
<th>2W/B; W/R</th>
<th>(B/d_{so})</th>
<th>Gravity level (g)</th>
<th>Porosity</th>
<th>Bond strength, (p_{b_s}) (N/m)</th>
<th>Number of disks</th>
<th>Average model run (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>0.12×0.24</td>
<td>0.008</td>
<td>30</td>
<td>8.9</td>
<td>1</td>
<td>0.2</td>
<td>2e⁷</td>
<td>174</td>
<td>343</td>
</tr>
<tr>
<td>Test 2</td>
<td>0.12×0.24</td>
<td>0.008</td>
<td>30</td>
<td>8.9</td>
<td>50</td>
<td>0.2</td>
<td>2e⁷</td>
<td>174</td>
<td>358</td>
</tr>
<tr>
<td>Test 3</td>
<td>0.12×0.24</td>
<td>0.008</td>
<td>30</td>
<td>8.9</td>
<td>100</td>
<td>0.2</td>
<td>2e⁷</td>
<td>174</td>
<td>406</td>
</tr>
<tr>
<td>Test 4</td>
<td>0.12×0.24</td>
<td>0.008</td>
<td>30</td>
<td>8.9</td>
<td>100</td>
<td>0.13</td>
<td>2e⁷</td>
<td>190</td>
<td>678</td>
</tr>
<tr>
<td>Test 5</td>
<td>0.12×0.24</td>
<td>0.008</td>
<td>30</td>
<td>8.9</td>
<td>100</td>
<td>0.13</td>
<td>1.0e¹²</td>
<td>190</td>
<td>678</td>
</tr>
<tr>
<td>Test 6</td>
<td>0.24×0.24</td>
<td>0.008</td>
<td>60</td>
<td>8.9</td>
<td>100</td>
<td>0.2</td>
<td>2e⁷</td>
<td>213</td>
<td>133</td>
</tr>
<tr>
<td>Test 7</td>
<td>0.24×0.48</td>
<td>0.016</td>
<td>30</td>
<td>17.8</td>
<td>100</td>
<td>0.2</td>
<td>2e⁷</td>
<td>695</td>
<td>649</td>
</tr>
</tbody>
</table>
Fig. 3. Unit base resistance, $q_{fb}$, under influence of (a, b) gravity level; (c) porosity; (d) parallel bond strength; (e) boundary effect; (f) scaled particle size effect. [Colour online.]
Fig. 4. Normalized (a) horizontal, (b) shear, and (c) vertical stresses measured at $r/R = 1.5$ in test 7. [Colour online.]

Fig. 5. Normalized (a) horizontal, (b) shear, and (c) vertical stresses measured at $y/R = 19.5$ in test 7. [Colour online.]
Fig. 6. Normalized stresses measured at relative positions to pile tip at various penetration depths in (a, d, g) test 7, derived from $(\sigma' - \sigma''_0)/q_b$; (b, e, h) test 7, derived from $(\sigma'q_0)/q_b$; and (c, f, i) test 3, derived from $(\sigma' - \sigma''_0)/q_b)$. [Colour online.]
prototype depth. Data between penetration depths of 24 and 176 mm in Fig. 3b were fitted linearly, with the ratio between the slopes of the lines from tests 1, 2, and 3 roughly equal to the ratio of their gravity levels. The figure is replotted in semi-log scale to be able to appreciate the trend of test 1. Data from the penetration depth less than 24 mm were not used for the linear regression analysis due to a different, shallow penetration mechanism (Jiang et al. 2006), and exclusion of these data was done to the linear regression in all the other subfigures of Fig. 3. The above results indicate the clear capability of the current DEM model in reflecting the in situ stress effect on the tip resistance behavior. The linear distribution of the tip resistance suggests a consistent penetration mechanism independent of the penetration depth.

The influence of the initial porosity of the sand is shown in Fig. 3c, where a reduction of the porosity from 0.2 to 0.13 leads to a 70% increase of the tip resistance at the final driven depth. The decrease of sand crushability via the increase of parallel bond strength results in a similarly clear increase of the tip resistance (Fig. 3d). In both cases, little difference of the tip resistance is observed for a prototype depth less than 11 m, above which the confining pressure is relatively low. It is worth noting that, under a low confining stress, the volumetric strains of the granular materials used in tests 3–5 are very close to each other, while under a high confining pressure, the material in test 4 contracts much less than the others (Fig. 2b).

The influences of the two model-scale variables W/R (where R is the pile radius) and B/dso, are given in Figs. 3e and 3f, respectively. It is found that doubling the model width causes little change of the tip resistance curve (Fig. 3e), indicating that the regular-sized model does not have a significant lateral boundary effect and the model width of 120 mm is enough to produce the physically sound and realistic penetration behavior. Interestingly, doubling the B/dso ratio produces a tip resistance curve (in test 7) that extends to a prototype depth twice that of a regular-sized test (e.g., test 3) and has a bilinear profile, whose first linear segment almost overlaps with that of test 3 and a second linear segment that has a slightly lower slope (Fig. 3f). This result has two implications: firstly, using a larger pile width does not change the tip resistance significantly within the low to moderate range of prototype depth (0–18 m), and the original pile width of 8 mm is enough for modeling purposes; secondly, for the larger penetration range of 18–36 m, the tip resistance curve exhibits a lower rate of increase, mainly due to the higher amount of particle breakage occurring around the pile tip.

**Pile penetration mechanism via stress normalization approach**

In the following section of the paper, it is endeavored to reveal a unique pile penetration mechanism by applying a stress normalization approach to the homogenized stress data obtained from the DEM simulations. This is motivated by the observed linear effect of in situ vertical stress on the tip resistance behavior and the attempt to establish a unique linkage between the tip resistance and penetration-induced stresses within the surrounding soil mass by removing the in situ stress effect. Such a goal is accomplished by presenting the stress analysis data in two parts: (i) normalized stress data measured on particle groups in this section and (ii) normalized full-field stress distribution in the next section.

**Group-based stress measurement**

As illustrated in Fig. 1b, a total number of 1800 particle groups, with a size of 0.5B × 0.5B for each group, are identified before penetration in tests 1–5 and test 7. Each particle group contains about 100 crushable agglomerates or uncrushable disks in test 7 and 25 in tests 1–5. The average stress in a group is found using a similar averaging procedure based on the measurement logic in PFC2D (Itasca Consulting Group 2008) and the position of each group is represented by the average coordinates of all particles within the group. It should be pointed out that the stress measurements were made on particle groups identified on the undeformed configuration prior to the pile penetration in this study, which is different from the stress measurements based on sampling windows defined on the deformed configuration in the previous study (Wang and Zhao 2014). This is because the current way of stress measurement allows a direct comparison with the experimentally measured stress data from the laboratory model pile test (Jardine et al. 2013b), in which the stress sensors were embedded within the sand mass before pile penetration.

**Synthesis of group-based stress measurement**

Stresses in the sand subjected to pile penetration consist of the initial stress and the stress change caused by penetration. The initial vertical stress in the field or centrifuge tests increases with depth, while remaining constant in calibration chamber tests with constant stress boundary conditions. The initial stress near the pile tip is relatively small compared with tip resistance and the impact of penetration decreases rapidly with increasing distance from the pile tip. A number of different methods (Jardine et al. 2013b; Yang et al. 2014) can be found in the literature to normalize the effective stresses (σ′) developed in sand during installation. If the stresses are normalized by the tip resistance (qt), after eliminating the influence of initial stress, they can be expressed by a 2D function of the normalized vertical and horizontal distances from the pile tip as

\[ \sigma' = \sigma'_{t} = f(h/R, r/R) \]

where \( \sigma'_{t} \) is the initial effective stress, \( h \) is the relative height from pile tip, \( r \) is the relative offset from the pile axis, and \( h/R \) and \( r/R \) are the normalized relative height from pile tip. A negative value of \( h/R \) means a position below the pile tip. In this way, the stress field within the sand mass becomes a function of the relative distance from the tip of a “stationary” pile, independent of the actual penetration depth of the tip.

The stress evolutions of particle groups whose initial positions are either on the same row or the same column (Fig. 1b) are tracked. Figure 4 shows the normalized stresses (\( \sigma'_{x1}, \sigma'_{y1}, \sigma'_{z1}, \sigma'_{x2}, \sigma'_{y2}, \sigma'_{z2} \))...
horizontal stress, shear stress, and vertical stress, respectively) of four groups of particles (i.e., G1, G2, G3, and G4) positioned at column 2, where \( r/R = 1.5 \), against the normalized penetration depth \( y/R \) (where \( y \) is the penetration depth from pile tip to the upper boundary) from test 7. It is seen that maxima of the normalized horizontal and vertical stresses developed, while the sign of the normalized shear stress changed as the pile tip leveled with the center of each group. The peak magnitudes of the normalized horizontal stress vary between 14.5% and 20.7%, show a similar level of variation to those of the normalized vertical stress, which vary between 9.2% and 13.6%. The similar pattern of results indicates that the normalized stresses vary as function of the normalized relative height from the pile tip.

The normalized stresses measured in four groups of particles (i.e., G2, G5, G6, and G7 in Fig. 1b) positioned at the same row with \( y/R = 19.5 \) (where \( y \) is the vertical position of a particle group from upper boundary) are shown in Fig. 5. One common feature is the

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**Fig. 8.** Normalized (a) horizontal, (c) vertical, and (e) shear stress contours (in %) during penetration in test 7; (b) radial and (d) vertical stress contours (in %) from experimental data (after Jardine et al. 2013b). [Colour online.]
rapid decrease of maximum stresses with the increasing distance from the pile tip, indicating the normalized stresses being a function of the relative distance from the particle group to the pile tip. The normalized stresses vary in the range of ±2% in “G6” and “G7”, which are positioned at r/R = 19.5 and r/R = 28.5, respectively. This observation indicates again that no severe lateral boundary effect is introduced in this model.

At various penetration depths, the normalized stresses of nine particle groups with initial positions fixed with respect to the pile tip (Fig. 1c) from tests 3 and 7 are plotted against y/R in Fig. 6. At any instance of penetration depth, the positions of any two groups are not on the same row or column so as to obtain a comprehensive and unbiased picture of the normalized stress distribution as a function of the relative distance from the particle group to the pile tip. The relationship between Δσy/R, and y/R (Figs. 6a, 6d, and 6g from test 7; Figs. 6c, 6f, and 6i from test 3) can be best represented as a constant function, which means the normalized stresses can be expressed as 2D, axially symmetric functions of the relative position to the pile tip. Note this is the case for all particle groups except “relG1”, which is immediately below the pile tip and not surprisingly shows some oscillations due to the very large deformation of the group. The higher particle number in each group in test 7 is seen to lead to a less scattered trend than test 3. Many authors (e.g., Yang et al. 2014) have used the empirical relationship (σ′ /σ0)(σ0 /pA)p13 to account for the dependence of σy on σ0, where σ′ is the initial vertical effective stress and pA is the atmospheric pressure. Figures 6b, 6c, and 6h show an example of an attempt to use this approach for stress normalization in test 7. It is clear that the normalized horizontal and vertical stresses of the groups away from the pile tip have a tendency to increase with penetration depth (Figs. 6b and 6h). This is because the tip resistance has a decreasing influence on the particle groups away from the pile tip and taking (σ′ /pA)p13 as a normalization factor underestimates the effect of initial vertical stress.

Besides the normalized stresses, it is also interesting to examine the trend of the mobilized friction angle of the particle groups. Figure 7 shows the variation of the mobilized friction angle against y/R from selected groups “relG1”, “relG4”, and “relG7” in test 7. The friction angle is calculated directly from arcsin[(σ′ /σ0)(σ0 /σ′)] where σ′ and σ0 are the major and minor principal stresses, respectively. The friction angle of “relG1” is seen to decrease from about 29° to 22° within a normalized driven depth of 3 to 9, and thereafter remains at a roughly constant value of 22°, which is close to the critical-state friction angle of DEM material “2±7-0.2” from the bi-axial simulation under the confining stress of 1.0 MPa. At “relG4” and “relG7”, the mobilized friction angles are much lower and exhibit much larger scatters, indicating that granular materials at these two positions have rarely reached the critical state.

**Full-field stress distribution**

Given the independency of the normalized stresses from the penetration depth seen in Fig. 6, it is now interesting to examine the contour map of the normalized stresses around a pile. The contour map is generated by calculating the average values of the normalized stresses of all particle groups at various penetration depths and then presenting the average full-field stress distribution around a pile with a generic penetration depth. Typical contour maps of the normalized horizontal, vertical, and shear stresses from test 7 are shown in Fig. 8. The position of each particle group is calculated as the average coordinates of all particles within the group at the deformed configuration. To demonstrate the validity of the simulation results, experimental data of the model pile from the calibration chamber experiments by Jardine et al. (2013b) are included in Fig. 8 for comparison. Although the normalization method of Jardine et al. (2013b) also takes the form of (σ′ /σ0)(σ0 /pA)p13, it provides the useful benchmarks for the simulation results, especially when the different initial stress conditions are taken into consideration.

Compared with the measured data of Jardine et al. (2013b), the normalized horizontal (Fig. 8a) and vertical stress (Fig. 8c) contour maps from test 7 show generally comparable patterns within a zone from the pile centerline to a horizontal offset about r/R = 10. Some divergence of the experimental results beyond this zone, especially near the boundaries, is observed. The difference is caused by different boundary conditions, which are the constant normal stress upper boundary with zero surcharge and fixed lower boundary in the numerical model, while there was a top and a bottom membrane applying a surcharge pressure of 150 kPa in the calibration chamber tests of Jardine et al. (2013b). The horizontal stress contour map shows a roughly symmetric distribution about the tip horizon, with the stress contour bulb extending to a horizontal distance of about 8R and decreasing rapidly to 3% at a vertical distance of 2R–3R vertically. In contrary, the vertical stress contour map shows an unsymmetric distribution about the tip horizon, with the stress contour bulb around the tip expanding almost downward vertically in the simulation, or in an inclined direction below the tip horizon in the experiment. Due to the complete elimination of the effect of initial stress, the normalized maximum horizontal and vertical stresses, which were 14% and 13%, respectively, are slightly lower than the experimental results. The normalized shear stress distribution displays a usual “X” pattern (Sheng et al. 2005), and can be divided into five zones, as shown in Fig. 8e, to allow a comparison with the sand deformation zones observed in the digital image correlation (DIC) study by Arshad and Jardine (2014). DIC is a noncontact image analysis method, based on grey-value digital images, which can determine the displacement and strain. In their study, the lower band of shear stress localization coincides with zone III, where the displacement field rotates from the vertical direction in zone I to the radial direction in zone IV. An additional zone V is identified between zone IV and the crushed band based on the observed upper shear band. The band also indicates that there are relative displacements between zones.
Fig. 9. Normalized stress contours (in %) during penetration in (a) test 3; (b) test 2; (c) test 4; (d) test 5. [Colour online.]

(a) 

(b)
Fig. 9 (concluded).
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This study endeavors to unravel a unique pile penetration mechanism in a homogeneous sand mass through an in-depth analysis of the stress field surrounding the pile from 2D DEM simulations. An invariant penetration mechanism revealed by the normalized stress fields eliminating the initial vertical stress effect was shown to be independent of the penetration depth. A new stress normalization method was adopted to synthesize the DEM-based stress data at different driven depths. The validity of the normalization method was verified by comparing the DEM simulation results with experimental data from calibration chamber tests, adopting an empirical normalization method. It was shown that the normalized horizontal and vertical stress contour maps from DEM simulations have generally comparable patterns to those from the experiments within a zone from the pile centerline to a horizontal offset about \( r/R = 10 \). Due to the different boundary conditions adopted in the simulations and experiments, some difference is observed near the lateral boundaries.

The normalized shear stress field showed incremental displacement zones similar to those observed in a previous DIC study. The identification of different shear stress zones provides insights into the general failure mechanism during the penetration process. Results from the parametric simulation study show that a lower initial porosity led to an up-moving trend of the maxima point of the stress contour and had a larger influence on the normalized shear stress field than other factors. The averaged \( \Delta \sigma^h_{b}/\Delta \sigma^v_{b} \) calculated at the “maximum” level from all the simulations showed a consistent linear pattern. The slopes of the linear correlations agree well with the measured gradients by Jardine et al. (2013), but fall between the ranges of the “spherical case” and “cylindrical case” predicted by the cavity expansion theory, suggesting the actual penetration mechanism is a mixed type of the two theoretical cases.

Finally, it needs to be mentioned that the current 2D model has some limitations, such as the absence of information about the circumferential stress and its influence on the pile tip resistance, and possibly larger lateral boundary effects, etc. These issues will be overcome in a three-dimensional simulation study that is currently underway using an open-source DEM code.

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**References**


