Discrete-continuum analysis of monotonic pile penetration in crushable sands

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Abstract: This paper presents numerical results from a two-dimensional discrete element method (DEM) simulation study on the monotonic pile installation in crushable sands. The particle breakage was included in the model by setting a crushable zone around a pile that was filled with parallel-bonded agglomerates. In the other part of the model, rigid, unbreakable particles are used to minimize the computational cost. Parametric studies were carried out to examine the effects of initial in situ vertical stress, soil void ratio, and particle crushability on the penetration resistance behavior. The validity of the DEM model was examined by comparing the simulation data with published results from laboratory centrifuge and calibration chamber tests on model pile installation. A variety of DEM analysis techniques were employed to make a detailed discrete-continuum study on the pile penetration mechanisms, particularly on the particle breakage-related soil mechanics incurred during the penetration process. Simulation results show that the in situ stress and particle breakage are the two competing factors dominating the tip resistance behavior. The underlying mechanism was elucidated through the analysis of stress and strain paths derived from pre-assigned sampling windows and their full-field distributions.

Key words: pile penetration, particle breakage, DEM simulation, discrete-continuum analysis, strain path.

Introduction

Despite being a subject under wide investigation over the last 50 years, the axial capacity of displacement piles in sand remains having large uncertainty in geotechnical engineering (Randolph et al. 1994). The complexity of the problem is created by many highly variable factors associated with the mechanical and geometrical properties of foundation soil and pile, and the engineering process of pile installation. Substantial research work has been dedicated to the exploration of fundamental mechanisms governing the pile–sand interaction during the pile-installation and pile-loading procedures, which is key to the development of more scientific, reliable, and economic methods for pile design (e.g., Randolph et al. 1994; Jardine and Chow 1996; Klotz and Coop 2001). However, many basic facets of the pile–sand interaction problem are still not clearly understood.

Apart from the possible high degree of inhomogeneity of foundation soil conditions, the primary technical difficulty associated with the formulation of pile penetration mechanism stems from the accurate and comprehensive account of the soil mechanics occurring in a special semi-infinite, boundaryless type of problem. A very relevant and important issue in the laboratory chamber test or centrifuge test of model piles, therefore, is to determine the minimum acceptable size of the testing apparatus that can reasonably reproduce the far field ground condition (e.g., Salgado et al. 1998). When the soil micromechanics is considered, the problem becomes much more convoluted as many scale-related (e.g., soil density and particle breakage) factors emerge into the picture.

Being aware of the above problems, a number of authors (e.g., Klotz and Coop 2001; White and Bolton 2004; White and Lehane 2004; Yang et al. 2010; Jardine et al. 2013) recently reported findings from a variety of high-quality centrifuge or calibration chamber tests of instrumented model pile installation, which greatly improved our understanding of the pile penetration problem. Advances were made in revealing the lateral stress distribution along the pile shaft and its evolution during the pile penetration process, as well as their variations as a function of initial soil state.
The discrete element method (DEM), which allows full access to the particle-scale kinetic and kinematic information, provides an alternative tool for the investigation of pile–soil interaction problem. Huang and Ma (1994) perhaps were the first to apply DEM to the pile penetration study and showed that the penetration behavior was affected by the loading history. Jiang et al. (2006, 2008) made detailed two-dimensional (2D) DEM analyses on the full-field kinematic variables of the granular ground due to the penetration of cone penetration testing (CPT). Recently, advances have also been made in three-dimensional (3D) DEM simulations of CPT. Using a virtual calibration chamber filled with a scaled granular equivalent of real sand, Arroyo et al. (2011) investigated the relative cone–chamber size effect on the cone resistance. A similar study was carried out by Lin and Wu (2012) on the dependence of penetration resistance on the penetrometer diameter. An effort that is in line with the size and scale problem of CPT was recently made by McDowell et al. (2012) to capture the realistic cone to particle size effect via a particle refinement method. However, in all the above DEM studies, particle breakage was not considered, therefore largely limiting the level of micromechanical insights obtained. A 2D DEM work that included particle breakage was reported by Lobo-Guerrero and Vallejo (2005), who modeled particle breakage by replacing a particle that meets the prescribed stress criterion with a few sub-particles and applied this method to the investigation of pile tip shape effects on the penetration behavior (Lobo-Guerrero and Vallejo 2007). However, in their studies, no effort was made to measure local stress and strain around the pile based on discrete data during the penetration process and limited discussion was given on the particle breakage-related micromechanics underlying the penetration behavior. As an alternative to DEM, Zhang et al. (2013) recently presented a rigorous approach to incorporate the effects of particle breakage into the formulation of the end-bearing capacity of piles based on the breakage mechanics theory (Elnav 2007a, 2007b).

Based on the above relevant DEM studies, this paper seeks to further exploit the potential of DEM in tackling the pile–sand interaction problem. The major thrust of this paper lies in an in-depth discrete-continuum analysis that reveals the stress and strain fields around and away from the pile during the penetration process, and thus provides a comprehensive picture of the penetration mechanism in crushable sands.

### Numerical model

#### 2D DEM model of pile–sand interaction

The DEM model is made up of a rectangular container filled with a well-compact, poly-dispersed assembly of round particles and a CPT-shaped model pile pushed gradually into the granular foundation at the middle of the box. Taking advantage of the model symmetry, only the right half of the model with a dimension of $15B (W) \times 30B (H)$ is used (Fig. 1a), where $B$ is the pile diameter and is equal to 8 mm. Both the bottom and right walls of the model are fixed. The bottom wall is set to be frictionless but the right wall is assigned a high friction coefficient of 0.9 to minimize any relative slip between the particles and the wall. The ratio of model to pile width is equal to 30 in our study, which agrees with the dimension ratios used in many experimental studies (e.g., Salgado et al. 1998; Bolton et al. 1999; White and Bolton 2004). It will be shown later that this dimension ratio is enough to eliminate the far-field boundary effects. The granular material is composed of rigid disks with diameters uniformly varying between 0.6 mm and 1.2 mm. To accommodate particle breakage in this study, we define a crushable zone with a size of $2B (W) \times 27B (H)$ surrounding the pile (Fig. 1a). In this crushable zone, particle breakage is allowed by the disintegration of agglomerates that are made of parallel-bonded particle clusters. The agglomerates used in this study are composed of 24–30 elementary balls with diameters between 0.069 and 0.278 mm (Fig. 1b). The particle cluster technique was frequently used by DEM researchers in the study of particle breakage behavior (e.g., Robertson 2000; McDowell and Harireche 2002; Cheng et al. 2004; Wang and Yan 2012, 2013) and has been demonstrated to be an effective and convenient tool for the investigation of micromechanics of crushable soils. Although in these previous studies this technique was used in 3D simulations, it would work equally well in 2D simulations as long as there is a large enough number of elementary balls in any agglomerate for breakage to occur. The physical parameters of rigid disks and crushable agglomerates used in the current study are shown in Table 1.

It needs to be mentioned that no intention exists in the current study to accurately simulate the breakage behavior of real sand particles using the above simplified technique, and the major purpose for including such a particle breakage behavior is to allow the investigation of the particle breakage effects on the pile penetration behavior that are mainly reflected through the material fabric change. To create a relatively uniform granular foundation, the multilayer under-compaction method (UCM) proposed by Jiang et al. (2003) was used to generate and compact the granular material. Specifically, five equal layers of granular materials are sequentially deposited and compacted to intermediate void ratios that are slightly lower than the target void ratio (Jiang et al. 2003). The compaction was carried out by moving downwards the top wall to attain a specific intermediate void ratio and after five times of...
such compaction the overall void ratio of the specimen will reach the target value. After the initial foundation is created, the crushable zone is introduced into the model by replacing all the rigid disks within this zone with agglomerates, each having the same diameter as the disk it replaces and is generated following the procedure described above. Then the whole foundation is reconsolidated under gravity until a complete equilibrium is achieved.

Apparently, an important issue in the above foundation generation process is the selection of the size of the crushable zone. A simple principle for this is that the size should be large enough to allow all the breakage-related physics that will affect the penetration behavior to be captured. However, a practical limitation for selecting a larger size of the crushable zone is the very high computational cost caused by a very large number of particles. For the currently adopted size of 218 x 278, the number of particles in the crushable zone is approximately 140 000, and approximately 170 000 in the whole sample. It will be shown that this size is sufficient for accommodating the vast majority of particle breakage events induced by pile penetration and can avoid any artificial, undesired effects along the interface between the crushable and uncushcrable zones.

The model pile is made up of two rigid wall segments, and at the bottom they formed a small cone making an angle of 60° with the horizontal. The wall friction coefficient is set to be 0.5 for the whole pile, simulating a rough pile–soil interface. After the granular foundation is prepared, the model pile is pushed monotonically into the foundation at a constant rate of 0.1 mm/s. The maximum penetration depth for the simulations in this study reached 25B.

**Parametric study**

We performed a total number of five simulations in this study to examine the influence of three model parameters, namely, initial vertical stress field, initial void ratio, and soil crushability. In particular, the initial vertical stress field was varied by employing an artificially raised gravity acceleration (e.g., 100g) that was similar to the physical effects attained in a centrifuge test, employing an artificially raised gravity acceleration (e.g., 100g). The initial vertical stress field, initial void ratio, and soil crushability are sized as 1 × 0.5B. Each sampling window is denoted using a code “C-R,” where C denotes column, R denotes row, a is the column or row number, and b is the window number in that column or row.

**Observation and analysis method**

To make a detailed discrete-continuum analysis of the soil mechanics occurring due to the pile installation, window-based and full-field stress and strain calculation techniques were employed to provide a comprehensive picture of the pile penetration mechanism. These techniques are briefly described below.

**Full-field stress and strain distributions**

The full-field shear and volumetric strain distributions will be generated using the meshless strain calculation method proposed and used by the first author in his previous studies (Wang et al. 2007a, 2007b; Wang and Gutierrez 2010; Wang and Yan 2013). The full-field stress distributions can be calculated using a grid-based method similar to the strain-calculation method. First, the iden-
ent levels of crushability and experimental data from direct shear tests on Dog’s Bay sand (DBS) by Tarantino and Hyde (2005). The direct shear simulations were conducted using the DEM model previously developed by the first author (Wang et al. 2007; Wang and Gutierrez 2010). It can be seen in Fig. 2 that under the same or similar confining pressure of 1 MPa, the overall stress ratio – strain curves from the DEM simulations lie below those from the direct shear tests. The critical-state friction angles given by the simulations for the low-crushability (i.e., with parallel bond strength value $2 \times 10^7$ N/m) sand and high-crushability (i.e., with parallel bond strength value $1 \times 10^7$ N/m) sand are about 24° and 22°, respectively, which are lower than the critical-state friction angle of about 33° given by the experiment for DBS. When the confining pressure is raised to 3 MPa, which is close to the maximum confinement level encountered in our pile simulations, the friction angles of the two kinds of materials are further reduced to about 21° and 16°, respectively. Although the shear strengths of the DEM materials are lower than that of a real crushable sand, the effects of different levels of DEM particle crushability on the pile penetration mechanism can be fully captured and demonstrated, as will be shown below. The simulated shear strengths of crushable sands are close to those obtained by Jiang et al. (2006) from biaxial simulations and used also in the 2D simulations of pile penetration in uncrushable sands.

**Tip and shaft resistance curves**

Figures 3 and 4 show the average pile tip resistance $q_b$ and shaft resistance $q_s$, respectively, throughout the penetration process from all the simulations. The results are categorized to show the effects of the three model variables on the penetration resistance in three sub figures. To demonstrate the validity of our simulation results, some measured data of model pile resistance from the centrifuge tests by Klotz and Coop (2001) are also included in Fig. 3 and Fig. 4 for comparison. It needs to be mentioned that the simulation data has been plotted using the equivalent prototype depth due to the employment of artificial gravitation fields that are intended to emulate the centrifuge test effects. It is seen in Fig. 3a that the three gravity levels used in the simulations, with all the other variables being the same, produce a unique $q_b$ curve against the prototype depth. The feature is also evident from the two selected sets of centrifuge test data on Leighton Buzzard sand (LBS) included in Fig. 3a, whose overall distribution is in good agreement with the simulation data although lower magnitudes of $q_b$ are found in the latter. However, when replotted against the actual model penetration depth, the simulation data are decomposed into three distinct curves, as shown in the inset diagram of Fig. 3a.

The result adequately verifies the effectiveness of the model to reproduce and reflect the in situ vertical stress field effects on the
penetration behavior. This element of the model’s capability is essential, because the in situ stress field is one of the crucial factors dominating the pile resistance behavior (Klotz and Coop 2001) and will be closely examined together with the soil crushability in this paper.

The tip resistance curves in Fig. 3 exhibit a nearly linear profile with little sign of a limiting tip resistance value at the end of the penetration process. Note these results are obtained in the low-crushability sand adopting the higher parallel bond strength of $2 \times 10^7$ N/m. Clear effects of the other two variables on the penetration resistance are illustrated in Figs. 3b–3c and 4b–4c. Prototype depth is again used in these plots, but it should be noted that no difference will arise if the actual model penetration depth is used because the same gravity level of $100g$ is used in these cases. The increase of soil crushability via the decrease of parallel bond strength results in an appreciable reduction of the tip resistance, especially at larger penetration depths. Interestingly, the nonlinear profile of the higher-crushability sands exhibits a trend of a constant value after about 8 m (see inset diagram of Fig. 3b). This feature can also be identified in the centrifuge test data of DBS included in Fig. 3b although the turning point marking the limiting tip resistance value may occur at a smaller depth.

To facilitate the comparison between the degrees of nonlinearity of the DEM data and centrifuge data of the tip resistance having different magnitudes, the data in Figs. 3a and 3b are replotted.
It is clear that the data of high-crushability sands from both DEM simulation and centrifuge test (i.e., DBS) exhibit much higher degrees of nonlinearity than those of low-crushability sands. Apparently the linear distribution of the tip resistance suggests a consistent penetration mechanism independent of the penetration depth that is dominated by the in situ stress field. This reconfirms the fallacy of adopting a limiting value of tip resistance for low-crushability sands in many design codes (Klotz and Coop 2001). In contrast, the higher nonlinearity with a trend towards a limiting value observed in a high-crushability sand is largely attributed to the massive amount of particle breakage caused by the pile penetration overriding the in situ stress effects. Detailed micromechanical evidences supporting the above statement will be presented later.

Fig. 6. Typical sampling window-based stress distributions throughout the penetration process from test 2: (a) vertical stress distributions from C-1 windows; (b) horizontal stress distributions from C-2 windows; (c) shear stress distributions from C-2 windows (counterclockwise is positive).

Fig. 7. Normalized vertical stress distributions from C-1 windows throughout the penetration process: (a) test 2; (b) test 4; (c) test 5.
Strong influence of the initial void ratio of the sand is also observed in Figs. 3c and 4c, where a reduction of the void ratio from 0.2 to 0.17 leads to a nearly 100% increase of both tip and shaft resistance values at any depth. Similar effects of void ratio are found in the centrifuge test data included in Figs. 3c and 4c. It is worth pointing out that this significant effect of soil relative density cannot be fully explained by the in situ stress field (i.e., local confinement). As shown in Fig. 5, the initial vertical and horizontal stress distributions from tests 2 and 4 are found to be very close. It is also unlikely to be caused by the model boundary effects, which will be demonstrated later. Indeed, the source of the relative density effect lies in the dense granular packing which could support the development of higher deviatoric stress.

Window-based stress distributions

Figure 6 shows the typical distribution of the average stress components from the column sampling windows at different pile depths. Normalized horizontal stress distributions from C-2 windows throughout the penetration process: (a) test 2; (b) test 4; (c) test 5.

Normalized horizontal stress distributions from C-2 windows throughout the penetration process: (a) test 2; (b) test 4; (c) test 5.

Test 2
Test 4
Test 5

Fig. 9. Normalized shear stress distributions from C-2 windows throughout the penetration process: (a) test 2; (b) test 4; (c) test 5 (counterclockwise is positive).

Normalized shear stress distributions from C-2 windows throughout the penetration process: (a) test 2; (b) test 4; (c) test 5.

Test 2
Test 4
Test 5

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driven depths from test 2. Only selected data of the average vertical stress in C-1 windows and average horizontal stress and shear stress in C-2 windows are presented because they are most closely related to the pile resistance behavior. For every stress distribution curve shown in Fig. 6, the peak value occurs approximately as the pile tip levels with the window centre. For the data in C-1 windows, each curve starts from the window immediately below the pile tip since the soil in all the windows above has been displaced (Fig. 6). A very large stress gradient is found to exist within a zone about \(4B \leq y \leq 6B\) beneath the pile tip, followed by a nearly constant profile extending close to the model bottom. Furthermore, the stress gradient and the subsequent constant value increase with the pile driven depth, with the latter reaching about 5 times the initial in situ vertical stress at a driven depth of 20B (Fig. 6). The envelope connecting all the peak points at various driven depths in Fig. 6a represents a close approximation of the \(q_o\) curve, although the envelope has slightly lower magnitudes due to the homogenization of the stress and strain made within the sampling windows. For the data in C-2 windows representing the very-near field adjacent to the pile shaft, the primary influence zone is found to extend 2–4B above and below the pile tip, with the values beyond the primary influence zone being much closer to the in situ values (Figs. 6b and 6c). No clear evidence of the degradation of radial effective stress or shaft resistance with the increasing penetration depth is spotted, as has been widely reported in field and centrifuge tests on instrumented piles (e.g., White and Lehane 2004) and attributed to the large number of load cycles imposed by pile driving or jacking, a factor that was not considered in this study.

To better demonstrate the effects of the variables of concern on the penetration mechanism, we show the stress distributions, which are normalized with respect to the corresponding initial in situ vertical stress at a driven depth of 20B (Fig. 6d). The envelope connecting all the peak points at various driven depths in Fig. 6a represents a close approximation of the \(q_o\) curve, although the envelope has slightly lower magnitudes due to the homogenization of the stress and strain made within the sampling windows. For the data in C-2 windows representing the very-near field adjacent to the pile shaft, the primary influence zone is found to extend 2–4B above and below the pile tip, with the values beyond the primary influence zone being much closer to the in situ values (Figs. 6b and 6c). No clear evidence of the degradation of radial effective stress or shaft resistance with the increasing penetration depth is spotted, as has been widely reported in field and centrifuge tests on instrumented piles (e.g., White and Lehane 2004) and attributed to the large number of load cycles imposed by pile driving or jacking, a factor that was not considered in this study.

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influence of void ratio and soil crushability by comparing test 2 to the other two tests respectively.

It can be found that the general profiles of the normalized stress ratios are similar to their corresponding stress profiles but the peak magnitudes of the normalized curves show much less variations throughout the penetration process, with the peak values of $\sigma'_{p}/\sigma_{p0}$ varying between 5 and 14, those of $\sigma'_{r}/\sigma_{r0}$ between 2 and 5, and those of $\tau'_{r}/\tau_{r0}$ between 0.4 and 0.8. The small variations of the peak normalized stress ratios suggest the dominant control of the in situ vertical stress on the penetration resistance behavior, with the void ratio and soil crushability being the interfering factors of this mechanism. Strong effects of the void ratio are again manifested in the overall much higher peak magnitudes of all the three groups of the normalized curves by comparing tests 2 and 4. Such a result can be well predicted using the normalized in situ stress ratio (with respect to the mean stress at the critical state) suggested by Klotz and Coop (2001).

The soil crushability effects can also be discerned from the overall much lower and gently decreasing peak values of $\sigma'_{p}/\sigma_{p0}$ with the increasing penetration depth from test 5 (Fig. 7c), which is consistent with the trend of a limiting tip resistance observed previously. Given the identical initial vertical stress distribution for tests 2 and 5, the attenuation of the in situ vertical stress effect is apparent and microscopically caused by a significant amount of particle crushing which hinders the development of local dilatancy and shear-induced anisotropy. This mechanism has been fully demonstrated in the fundamental studies on the mechanical behavior of crushable sands conducted previously by the first author (Wang and Yan 2012, 2013). Little work in the literature, however, is known to the authors that attempt to explain the linkage between soil crushing and pile tip resistance from the microscopic point of view.

More direct information of particle crushing from the simulations is given in Fig. 10, in which the survival rate distribution of agglomerates and the evolution of particle size distribution (PSD) within the crushable zone from tests 2 and 5 are shown. The survival rate is calculated as the ratio of the number of existing parallel bonds to the total number of ball contacts per unit area. It is interesting to find in Figs. 10a and 10b that three or four zones with increasing values of survival rate are developed from the pile shaft towards the radial direction. This distribution is very similar to the one described by Yang et al. (2010) inferred from the experimental observations, where the three zones with different intensity levels of shearing are distinguished using the visual observation and the capillary suction-adherence criterion, the latter being a direct reflection of the change of the average inter-grain pore size as a consequence of significant particle breakage. It is noted that all the crushing zones start from beneath the pile tip and extend over the entire pile shaft. The particle crushing mostly took place in front of the pile tip then the fragments migrated to the side and coated the pile shaft, which is identical to the results obtained by Lobo-Guerrero and Vallejo (2005). However there is a trend of decreasing thickness of the high crushing zone (i.e., zone 1) away from the tip, which is opposite to the experimental trend (Yang et al 2010). This is probably again due to the absence of intensive cyclic shearing along the shaft in our simulations, reflecting only the particle crushing induced by the pile tip compression.

Figures 10d and 10e show the evolutions of PSD within the entire crushable zone from tests 2 and 5. It is seen that the PSD curves stay concave upwards and rotate around the maximum particle size. Due to the higher particle crushability, the PSD in test 5 approaches closer to the linear distribution in the double logarithmic graph, suggesting a trend towards the fractal distribution (McDowell and Bolton 1998). However, the PSD does not seem to converge towards a stable grading in either case. It should be noted that, although the current numerical study is conducted in 2D, the particle-cluster technique employed for the breakage simulation is fully effective in producing a meaningful result of PSD evolution that is comparable to that from a previous 3D DEM study employing the same technique (Wang and Yan 2013).

Window-based strain paths
Next, we present simulation data of strain path from the sampling windows. Figure 11 shows the typical evolutions of total natural horizontal and vertical strains calculated within C-2 windows from test 2. It needs to be mentioned that the natural strain was particularly used in this and the next three figures to permit a direct comparison with the PIV results of White and Bolton (2004). The procedure to calculate the natural strain and rotation tensors is identical to that of White and Bolton (2004).

It is evident in Fig. 11 that similar patterns hold for the strain profiles from windows at various elevations: the horizontal strain paths are featured with an initial extension followed by a sharp reversal into a strong compression and then a quasi-constant regime; whereas the vertical strain paths are featured with an initial compression followed by a sharp reversal into a strong extension and then a quasi-constant regime too. Both strain reversals occur when the pile tip levels with the window centre, echoing the sharp unloading event of the soil element from that moment. The
Strain and rotation paths from strain path retrieval windows in test 2; for all strain components, compression is positive; for rotation, counterclockwise is positive (from White and Bolton 2004).

Final constant strain values after the passing of the pile tip are also found not to vary much, agreeing with the experimental observations of White and Bolton (2004). The above results imply again a consistent penetration mechanism from the soil deformation point of view and verify the kinematic constraint condition (i.e., zero strain increment) beside the pile shaft. They also serve as the basis of the strain path analysis approach taken by White and Bolton (2004), in which the penetration process is treated as the flow of a soil element located at any elevation towards the tip of a stationary pile.

Figures 12–14 illustrate the strain paths from selected simulations obtained using the same approach of White and Bolton (2004). Specifically, three strain path retrieval windows at the same depth of 12B were chosen to produce all the strain path data (Fig. 1c). Comparisons to their typical measured data on Leighton Buzzard sand and Dog’s Bay sand are also included in Figs. 12 and 13, respectively. In each figure, data of soil elements in the very near field, near field, and far field are presented with the corresponding ratios of the distance between the soil element and the pile centreline to the pile radius, \(x_B/\text{B}\), equal to 2, 4, and 10, respectively, from our simulations, and the corresponding ratios from White and Bolton (2004) are labeled in Figs. 12 and 13; and \(2h/\text{B}\) denotes the normalized vertical distance between the soil element and the pile tip.

It is seen in Figs. 12 and 13 that all the profiles of horizontal strain, vertical strain, and rotation have overall good agreement with the experimental data of White and Bolton (2004). The main divergence appears to be the much later development of the soil strain in the very near and near fields in our simulations. The authors assume that the smaller stress influence zone ahead of the pile tip (Figs. 7 and 8) is caused by the large gravity fields implemented in the simulations. However, the reproduction of the stress and rotation paths on the qualitative level, particularly the marked strain reversal behavior, attests to the model’s capability to simulate the critical soil deformation behavior before and after the passing of the pile tip.

The soil mechanics at various distances from the pile shaft are found to be affected strongly by soil crushability and initial void ratio by comparing Figs. 13 and 14 to Fig. 12. In test 2 with the high void ratio and low particle crushability, an initial very slight volumetric compression followed by a strong dilation when the pile tip is within a distance of 5 pile radii, and then a stronger compression after the passing of the tip is observed in the very near field (Fig. 12a). When the particle crushability is high, there is an absence of dilation before the passing of the tip, followed by an abrupt, severe turn into the compression due to a massive amount of particle breakage (Fig. 13a). These results also agree with the observation by White and Bolton (2004) on the less crushable
Leighton Buzzard sand and highly crushable Dog’s Bay sand. It is worth pointing out that the volumetric strain reversal seen in test 2 is indeed caused by the rotation of the principal stress (i.e., from vertical to horizontal) as the soil element passes the tip. The subsequent unloading event continues in both processes of the volumetric compression and principal stress rotation until the critical state with constant volume is reached. For test 4 with the low void ratio, the strain reversal is much less pronounced, making the entire volumetric strain path a roughly constant profile. This implies that the soil element does not deviate significantly from the critical state throughout the process due to the combined effects of initial higher confinement and more significant particle breakage (Fig. 14).

The overall pattern of the volumetric strain behavior in the near field is similar to that of the very near field but less pronounced due to the reduced stress field (Figs. 12b, 13b, and 14b). In the far field, the strain reversal phenomenon almost disappears in each test (Figs. 12c, 13c, and 14c).

Model boundary effects

It is necessary to demonstrate the model boundary effects in our study. From Figs. 12c, 13c, and 14c, it is already seen that the strains are reduced to a very low level at \(2x_0/B = 10\). Since soils are uncrushable in this region, the initial void ratio dominates the volumetric strain behavior, with slight compression and dilation found in tests 2 and 4, respectively. More information about the boundary effects is given by the distributions of normalized horizontal stress from the row windows up to the lateral boundary, as shown in Fig. 15. It should be mentioned that the normalization in Fig. 15 is made with respect to the initial in situ horizontal stress to better depict any boundary effect. It is found that at the two selected penetration depths of 5B and 10B, the normalized horizontal stress at the elevation of the pile tip decays rapidly to a nearly constant value of 2 or less in a distance of approximately 7–8B. The stress distributions from the row windows above and below the pile tip have all constant profiles with values less than 2. This observation indicates further that all the pile-induced soil mechanics has been covered by the current model and no severe lateral boundary effect is introduced.

Full-field stress and strain distributions

Finally, we present the full-field shear stress and incremental shear strain and volumetric strain distributions, the typical examples of which are shown in Figs. 16 and 17. The shear stress data plotted were the deviatoric stress \(\tau\) of the plane-strain stress tensor, which was also normalized with respect to the initial in situ
vertical stress $\sigma_{zz}$ as done in Fig. 9. Since the current problem involves extremely large strain in soils near the pile shaft and around the pile tip, the currently deformed ground, instead of the initial ground configuration (before pile installation), was used as the reference configuration for the calculation of incremental shear strain ($\Delta \varepsilon_{xy}$) and volumetric strain ($\Delta \varepsilon_{v}$) incurred within the next pile advancement of $B$ to obtain a better picture of the local soil deformation around the tip. It is first interesting to find that two inclined bands of shear stress localization emanate from the pile tip towards lower- and upper-right, respectively, both making an angle of about 45° from the horizontal, especially at a larger penetration depth. The direction of emanation apparently marks the principal stress direction around and away from the pile tip. The opposite signs of the shear stress in the two bands are consistent with the principal stress rotation across the tip, which causes the strain reversal phenomenon. The high stress zone is concentrated around the tip and decays gradually along the direction of emanation. As compared to test 2, the high crushable soil in test 5 results in less pronounced emanation bands of localized stress, particularly below the pile tip (Figs. 17a and 17b), suggesting again the attenuation of the in situ vertical stress effects due to massive particle breakage.

From the plots of shear strain distribution, it is found that the major zone of strain localization extends from the pile tip to a distance of approximately $5B-7B$ in the radial direction and upwards along the pile shaft as well (Figs. 16c, 16d and 17c, 17d). Similar patterns of strain localization featured with a “nose cone” beneath the tip and several shear bands with opposite signs of shear strain curving from below the pile tip around to the upward direction are observed at two different penetration depths (Figs. 16c and 16d). Such a pattern exhibits both features of cavity expansion (Vesic 1972; Yu and Houlsby 1991; Salgado et al. 1997) and bearing capacity-type failure, the latter failing to be observed by White and Bolton (2004), probably because the incremental displacement and (or) strain was not used in their study. For the case of high-crushable soil (i.e., test 5), the feature of bearing capacity failure is less pronounced because the significant particle breakage greatly reduces the soil dilation (Figs. 17c and 17d). Such a trend, however, also exists in the case of low-crushable soil (i.e., test 2) as the penetration depth becomes larger, resulting in more particle breakage as well. The volumetric strain fields from both tests exhibit similar patterns to those of the shear strain fields, with the high volumetric contraction zones fully covered by the crushable zone (Figs. 16e, 16f and 17e, 17f). A smooth transi-
tion across the boundary between the crushable and uncrushable zones is found. This observation, together with the results shown in Figs. 10a and 10b, indicate the adequate effectiveness and validity of the model setting of the crushable zone.

**Concluding remarks**

This paper reexamines the problem of close-ended pile installation in crushable sands using the discrete element simulation method. Through the careful construction of the pile–sand interaction model and the application of a series of DEM analysis techniques, we presented a detailed and in-depth discrete-continuum study on the pile penetration mechanism, elucidating the effects of and interplay between the in situ vertical stress field, soil crushability, and void ratio on the penetration behavior. By comparing the simulation results with published experimental data of model pile installation in centrifuge and calibration chamber tests, we demonstrated the high capabilities of the model in reproducing many facets of the soil mechanics during the installation process seen in the laboratory or field, particularly the dominant influence of the in situ vertical stress on the tip resistance in a low-crushability sand, the competition between particle breakage and in situ stress in controlling the tip resistance as the soil crushability increases, as well as the essential strain reversal phenomenon in a soil element flowing past a “stationary” pile. It should be stressed that achieving the understanding of these mechanisms is not affected by the quantitative disparities between the simulation and experimental data, which is resulted from the simplification of the reality by the current model and the imperfect model validation process. No significant boundary effects due to the current plane-strain model employing rigid disks, based on the comparison with the plane-strain experimental results, were found. This is probably because the current DEM materials exhibited an overall lower strength than the real sands.

Attributed to the unique capability of DEM in providing all particle-scale data, this paper offers rich insights into the micro-mechanical processes and mechanisms underlying a range of penetration behavior that was known to be scientifically and practically important but not fully understood yet. Notable examples include the semi-constant tip resistance profile at larger penetration depths in high-crushability sands, which is caused by significant particle breakage prevailing against the in situ stress, and the strain reversal behavior after the passing of the tip, which is caused by the principal stress rotation. These and many other novel insights have advanced our understanding of the pile monotonic penetration mechanism in crushable sand.

However, due to the absence of the cyclic loading along the pile shaft that commonly occurs in the dynamic pile driving process and the inclusion of soil creep behavior in our simulations, some important aspects of penetration behavior cannot be captured, including shaft friction fatigue, densification of an intensive shearing zone along the shaft, pile aging, and radial relaxation.
Fig. 16. Full-field stress and strain distributions from test 2: (a and b) normalized shear stress ($\tau'/\tau_n$) distributions at (a) $D = 5B$ and (b) $D = 15B$; (c and d) incremental shear strain ($\Delta \epsilon_{xy}$) distributions at (c) $D = 5B$ and (d) $D = 15B$; (e and f) incremental volumetric ($\Delta \epsilon_v$) strain distributions at (e) $D = 5B$ and (f) $D = 15B$ (for shear stress and shear strain, counterclockwise is positive; for volumetric strain, contraction is positive).
Fig. 17. Full-field stress and strain distributions from test 5: (a and b) normalized shear stress \((\tau/x_B)\) distributions at (a) \(D = 5B\) and (b) \(D = 15B\); (c and d) incremental shear strain \((\Delta\varepsilon_{xy})\) distributions at (c) \(D = 5B\) and (d) \(D = 15B\); (e and f) incremental volumetric strain \((\Delta\varepsilon_v)\) distributions at (e) \(D = 5B\) and (f) \(D = 15B\) (for shear stress and shear strain, counterclockwise is positive; for volumetric strain, contraction is positive).
These issues are currently under investigation using an improved model that incorporates soil creep and aging effects.

Acknowledgements

The study presented in this article was supported by the General Research Fund CityU No. 122813 from the Research Grant Council of the Hong Kong SAR and Research Grant No. 5109182 and 51379180 from the National Science Foundation of China.

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